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## SUMMARY OF RESEARCH

The major goals of this research are to generate a deep understanding of communication channels and sources and to use this understanding in the development of reliable, efficient communication techniques. Research has centered on four types of channels: additive Gaussian noise channels, fading dispersive channels, atmospheric optical channels, and, more abstractly, discrete memoryless channels.

### 1. Additive Gaussian Noise Channels

Much of the recent effort on Gaussian noise channels has been concerned with using the experimental sequential decoding facility to simulate the effect of sequential decoding on a deep space link. Early simulations assumed the availability of both phase reference and synchronization.<sup>1</sup> It was found that reliable communication could be achieved by using an encoder constraint length of 18 or 24 information bits and a signal-to-noise ratio 3 db or 4db above channel capacity. The results on the distribution of computation and on error probability were in excellent agreement with theory. More recent simulations have employed a decision-directed feedback technique within the decoder to provide a channel phase reference in the presence of various kinds of random phase variations.<sup>2</sup> A general theoretical investigation is under way for obtaining synchronization in communication systems with the use of coding.

An investigation has been completed on the topic of optimum receivers for Markov sources with nonlinear modulation on a Gaussian noise channel.<sup>3</sup> Approximate mean-square-error estimators and the corresponding receiver structures were derived by using a state-variable approach.

### 2. Fading Dispersive Channels

Although bandlimited fading dispersive channels are often encountered, neither their ultimate performance limitations nor effective techniques for communicating over them have been well understood. These problems have been attacked from a variety of standpoints in the last six months. One doctoral thesis, which should be completed by January 1967, is concerned with the ultimate performance limitations, and has reduced the problem

to a form in which it can be solved by a combination of analytical and numerical techniques. A second approach is to represent the channel as a set of simpler parallel channels, all of which are statistically related. A doctoral thesis is under way to investigate the general properties of parallel, statistically related channels in the absence of crosstalk. A third approach, which is being pursued in a prospective doctoral thesis, is to study dispersive channels in the limit of very slow time variations.

Experimental study of fading dispersive channels is also being pursued, with the use of the "bubble-tank" channel. Instrumentation and computer programs have been developed for this channel and some coding techniques have been investigated experimentally.<sup>4</sup>

### 3. Atmospheric Optical Channels

An experimental and theoretical investigation of the turbulent atmosphere as an optical communication channel has been initiated. The experimental aspects of the investigation have been pursued by three graduate students who are developing equipment to measure the phase, intensity, and polarization fluctuations induced by the turbulence; their work should be completed by the middle of September. The theoretical investigation, which began last fall, has focused upon an analysis technique involving geometric optics; preliminary results were reported at the recent Optical Society of America meeting and preparation of a paper has begun.<sup>5</sup>

### 4. Discrete Memoryless Channels

A paper has been completed<sup>6</sup> giving lower bounds to the minimum achievable error probability when using block coding over general discrete memoryless channels. The exponential dependence on block length for these bounds agrees at zero transmission rate and at high transmission rates with previously derived upper bounds. For source rates above channel capacity, rate-distortion techniques have been used to find exact expressions for the minimum error probability achievable for discrete memoryless sources.<sup>7</sup> Next, a number of bounds have been derived on the minimum probability of error achievable when feedback is available from receiver to transmitter. Finally, an investigation is almost completed

on the effect of running a number of sequential decoders in parallel with simple algebraic coding between them. Such a technique allows reliable communication at rates above the normal computational cutoff rate of sequential decoding.

#### References

1. D. D. Falconer and C. W. Niessen, "Simulation of Sequential Decoding for a Telemetry Channel," Quarterly Progress Report No. 80, Research Laboratory of Electronics, M. I. T., January 15, 1966, pp. 183-193.
2. R. Gray, "Simulation of Sequential Decoding with Decision Directed Channel Measurements," S. M. Thesis, Department of Electrical Engineering, M. I. T., June 1966.
3. D. L. Snyder, "The State Variable Approach to Continuous Estimation," Sc. D. Thesis, Department of Electrical Engineering, M. I. T., June 1966.
4. J. C. Molden, "High Rate Reliability over Fading Dispersive Communication Channels," S. M. Thesis, Department of Electrical Engineering, M. I. T., June 1966.
5. R. S. Kennedy and E. V. Hoversten, "A Model of Propagation through a Turbulent Atmosphere for Optical Communication," Fiftieth Meeting, Optical Society of America, Washington, D. C., March 1966.
6. C. E. Shannon, R. G. Gallager, and E. R. Berlekamp, "Lower Bounds to Error Probability for Coding on Discrete Memoryless Channels" (submitted for publication to Information and Control).
7. J. T. Pinkston, "An Application of Rate-Distortion Theory to a Converse to the Coding Theorem" (submitted for publication to IEEE Transactions on Information Theory).

#### Publications

- D. Chase, "Truncating a Convolutional Code," Quarterly Progress Report No. 80, Research Laboratory of Electronics, M. I. T., January 15, 1966, pp. 180-183.
- R. Pilc, "Coding for Source-Channel Pairs," Quarterly Progress Report No. 81, Research Laboratory of Electronics, M. I. T., April 15, 1966, pp. 169-173.
- D. D. Falconer, "An Upper Bound on the Distribution of Computation for Sequential Decoding with Rate above  $R_{comp}$ ," Quarterly Progress Report No. 81, Research Laboratory of Electronics, April 15, 1966, pp. 174-179.

- G. David Forney, Jr., "Concatented Coding," Technical Report 440, Research Laboratory of Electronics, M.I.T., December 1, 1965.
- C. W. Niessen, "An Experimental Facility for Sequential Decoding," Technical Report 450, Research Laboratory of Electronics, M.I.T., and Technical Report 396, Lincoln Laboratory, M.I.T., September 13, 1965.
- C. J. Boardman and H. L. Van Trees, "Optimum Angle Modulation," IEEE Trans., Vol. COM-13, No. 4, pp. 452-469, December 1965.
- H. L. Van Trees, "Analog Communication over Randomly Time-Varying Channels," IEEE Trans. on Information Theory, Vol. 12, No. 1, pp. 51-63, January 1966.
- D. L. Snyder, "Some Useful Expressions for Optimum Linear Filtering in White Noise: II," Proc. IEEE (Letter), pp. 1254-1255, September 1965.
- D. L. Snyder, "Optimum Linear Filtering of an Integrated Signal in White Noise," IEEE Trans., Vol. AES-2, No. 2, pp. 231-232, March 1966.